FNAL, Technical Division Aug. 08, 2005
Note # TD-05-037

Proton Driver Front End.
Warm Section Focusing Solenoid

I. Introduction

Focusing solenoids are used for beam focusing in the front part of the Proton Driver to reduce beam halo. There are two parts of the Front End that use warm (not superconducting) accelerating sections: Drift Tube Linac (DTL) and Middle Energy Beam Transport (MEBT) section with beam chopper. These sections do not require thorough screening of magnetic field, so simple solenoids can be used there. During the initial phase of the Front End design, a major set of focusing solenoid parameters was chosen to start with. These parameters are:

- maximal solenoid effective strength: FI = $\int B_c^{\,2} \cdot dl$ = 2.64 T^2m;
- maximal length (including cryostat) : L_{max} = 180 mm;
- beam pipe bore diameter : D_{bp} = 25 mm;

Concept of the solenoid design is described elsewhere [1], and main design features are shown in the picture in Fig. 1:

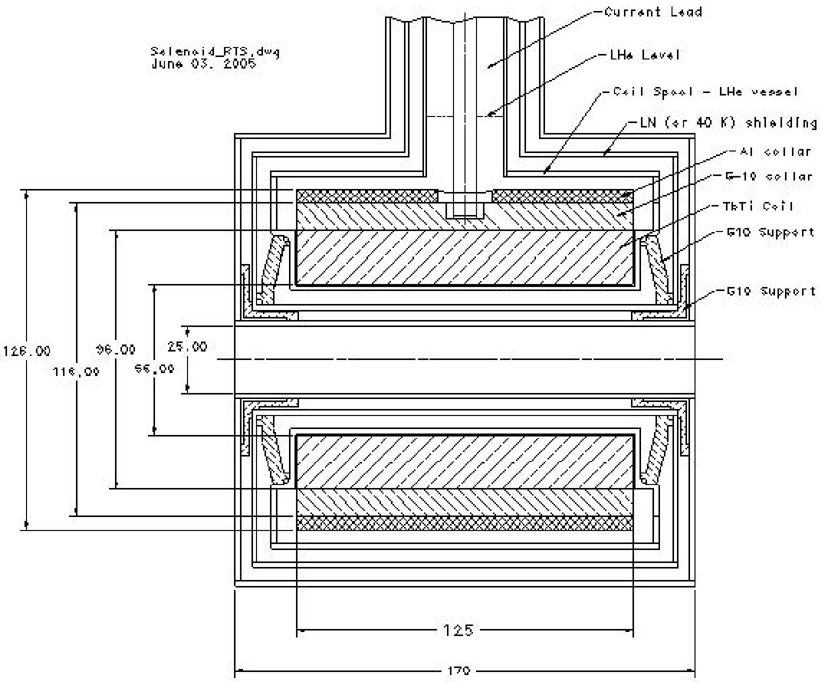

Figure 1: Solenoid: main features

This configuration of the solenoid was found by using magnet modeling program, without much of optimization applied. Nevertheless, simple configuration of the coil in the solenoid allows using some common means of analysis in attempt to find better configuration or to see possible limitations. This analysis is a scope of this note.

II Study algorithm

For superconducting solenoids, wire critical current is the main limiting factor on the road to higher magnetic fields. First prototypes of solenoids are to be built using available 0.808 mm diameter NbTi wire. Although the strand properties are to be measured yet, at the stage of modeling, critical current density was chosen close to what was specified for SSC strand or to the performance of the best industrial scale heat treated composite [2]. Assuming Cu to non-Cu ratio of the strand of 1.3, this current density can be translated into wire critical current, which is shown by blue line in Fig. 2 below as a function of magnetic field.

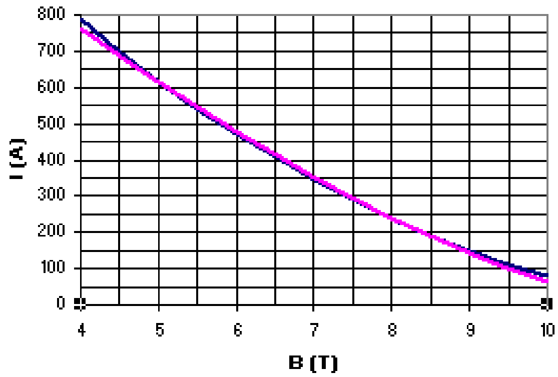

Fig. 2: Critical current of an SSC-type, 0.808 mm, NbTi wire.

Red curve in the figure gives an analytical representation of this curve made in the form:

$$I_{cr}(B) = 613 - 145\cdot(B-5) + 7\cdot(B-5)^2 \qquad /1/$$

In the range from 5T to 9 T this approximation is within 1.5% from the declared curve.

Because the effective strength is a major parameter, we will need an analytical expression of magnetic field along the axis of the solenoid to integrate it along the length. Figure 3 provides a simplified picture of a solenoid showing symbols used for this study.

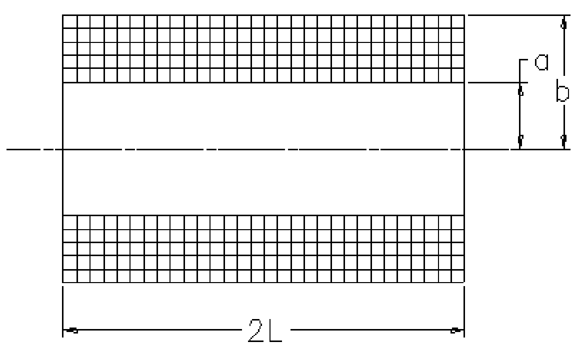

Fig. 3. Thick solenoid coil

Before writing the expression, which is widely used (e.g., see [3]), let's define a compaction factor *k* as a ratio of the total cross-section of NbTi wire (copper plus non-copper) and find the number of turns in the coil:

$$N = S_{coil} / S_w \qquad /2/$$

Here $S_{coil} = 2a^2\beta(\alpha\text{-}1)$, with $\alpha = b/a$, $\beta = L/a$, and $\boldsymbol{a}$ is the inner radius of the coil (Fig. 3). S_w is the effective cross-section area of the insulated wire (taking into the account interlayer insulation):

$$S_w = S_b/k$$

where S_b is cross-section area of wire without insulation (bare wire).
So, finally,

$$\mathrm{N} = 2a^2\beta(\alpha\text{-}1)k / S_b. \qquad /3/$$

Now we can write down:

$$H(z') = \frac{NI}{4L}\left[\frac{\beta - z'}{\alpha - 1}\ln\frac{\alpha + \sqrt{\alpha^2 + (\beta - z')^2}}{1 + \sqrt{1 + (\beta - z')^2}} + \frac{\beta + z'}{\alpha - 1}\ln\frac{\alpha + \sqrt{\alpha^2 + (\beta + z')^2}}{1 + \sqrt{1 + (\beta + z')^2}}\right] \qquad /4/$$

Here z' is the length along the axis normalized by the inner radius a: $z' = z/a$.

To calculate maximal field in the coil, which differs from the field on the axis, we will need an analytical expression for the magnetic field in the center plane of the solenoid. In this plane, magnetic field is always parallel to the axis. Fig. 4 helps to visualize the geometrical relationships that were taken into the account.

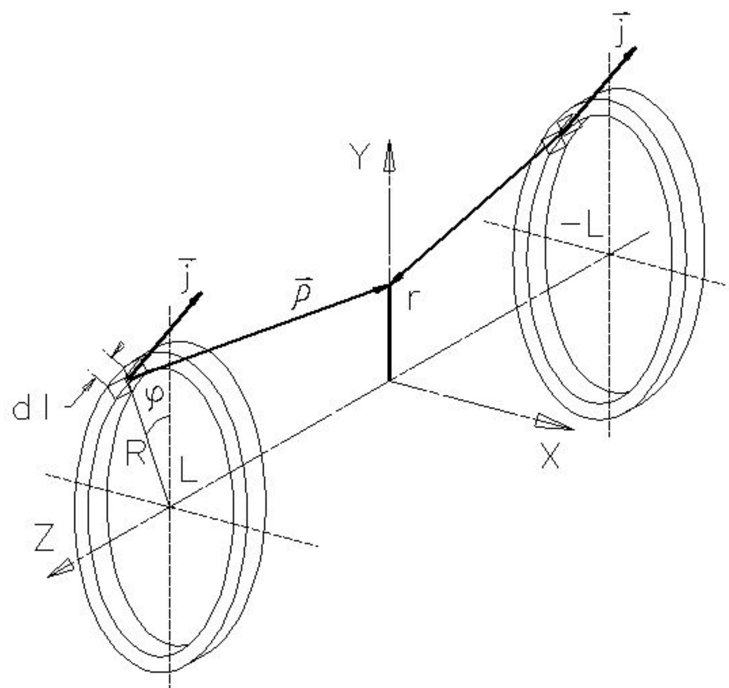

Fig. 4: Understanding magnetic field in the central plane of a solenoid.

Coordinate system is positioned in the center of the coil. Two current rings within the coil are located symmetrically relative to the XY plane at the distance ±z. Because of the azimuthal symmetry, it is enough to find field at any azimuthal position, so, we will look for it at the point with coordinates (0, r, 0). Ampere's law will be used for this exercise that states that the magnetic field can be found if we know distribution of currents and distances from partial sources to the point of observation:

$$\vec{H}_P = \frac{1}{4\pi}\cdot\int\frac{\vec{j}_Q \times \vec{\rho}_{QP}}{\rho_{QP}^3}dV_Q \qquad /5/$$

where Q is the source point (variable) and P is the observation point.

For the geometry shown in Fig. 4, $dV = R \cdot d\varphi \cdot dR \cdot dz$ and the average current density can be found if we know the wire current I, coil dimensions a,b,and L, and compaction factor k. From the geometry in Fig. 4 is clear that the components of magnetic field along X and Y from the two symmetrical parts of the rings cancel each other. The components along Z have the same sign and do not depend on the sign of the angle φ.

Vector $\vec{j}$ has the next coordinates:

$$\vec{j} \equiv \frac{N \cdot I}{2 \cdot a^2 \cdot \beta \cdot (\alpha - 1)} \cdot \begin{pmatrix} \cos\varphi \\ -\sin\varphi \\ 0 \end{pmatrix} \qquad /6/$$

Coordinates of vector $\vec{\rho}_{QP}$ are:

$$\vec{\rho} \equiv \begin{pmatrix} -R \cdot \sin\varphi \\ r - R \cdot \cos\varphi \\ -z \end{pmatrix} \qquad /7/$$

Leaving only z-component of the vector $\vec{j}_Q \times \vec{\rho}_{QP}$, we can write:

$$\left(\vec{j}_Q \times \vec{\rho}_{QP}\right)_Z = \frac{N \cdot I}{2 \cdot a^2 \cdot \beta \cdot (\alpha - 1)} \cdot (R - r \cdot \cos\varphi) \qquad /8/$$

Knowing components of the vector $\vec{\rho}$ above, its module can be easily found:

$$\rho^2 = R^2 + r^2 + z^2 - 2 \cdot R \cdot r \cdot \cos\varphi \qquad /9/$$

What is left is to take the integral to find magnetic field in the center plane:

$$B(r) = \frac{\mu_0}{2\pi} \cdot \frac{N \cdot I}{a^2 \cdot \beta \cdot (\alpha - 1)} \cdot \int_0^{\pi} d\varphi \int_0^{L} dz \int_a^{b} R \cdot \frac{R - r \cdot \cos\varphi}{\left(R^2 + r^2 + z^2 - 2R \cdot r \cdot \cos\varphi\right)^{3/2}} dR \qquad /10/$$

To make variables of this expression dimensionless, we will normalize them to the inner radius a (R’ = R/a, r’ = r/a, and Z’ = z/a) to have finally:

$$B(r') = \frac{\mu_0}{2\pi} \cdot \frac{N \cdot I}{a \cdot \beta \cdot (\alpha - 1)} \cdot \int_0^{\pi} d\varphi \int_0^{\beta} dz' \int_1^{\alpha} R' \cdot \frac{(R' - r' \cdot \cos\varphi) \cdot dR'}{\left(R'^2 + r'^2 + z'^2 - 2R' \cdot r' \cdot \cos\varphi\right)^{3/2}} \qquad /11/$$

Now we know everything to make a parametrical study of a focusing solenoid.

<u>The computation steps are outlined below:</u>

1. Initial parameters are the inner radius of the coil a and the compaction factor k.
2. Taking a pair of dimensionless parameters α and β, we can immediately find volume of the coil V using /xx/.
3. Taking some arbitrary sample current I and using /11/ for r’ = 1, we find efficiency of the solenoid (that is maximal magnetic field on the wire corresponding to this current I). Knowing that this field in a linear function of the current, we can find analytically intersection of the magnet load curve with the wire critical current diagram /1/.
4. Now we know maximal achievable field in the coil B_{cr} and corresponding current in wire I_{cr}. Central magnetic field B_{cent} can also be calculated knowing this current.

5. Knowing maximal current, field integral *FI* can be found using analytical expression for the magnetic field /4/
6. As a result, we have a set of data tables for *FI*, B_{cent}, B_{cr}, I_{cr}, and V to analyze.

III. Data Analysis

The algorithm described above was realized using MathCad environment for solving equation and finding integrals (see attachment for the MathCad file script).

Geometrical parameters of solenoid coil were scanned to cover rather wide range of the coil shapes: α ∈ (1.5 ÷3), β ∈ (1 ÷3). Corresponding tables of results shown in Fig. 5.

a = 25 mm, k = 0.563

FI (β = 2.6)

α \ β	1	1.5	2	2.5	3
1.5	1.35	2.285	3.265	4.27	5.295
2	2.173	3.472	4.834	6.23	7.659
2.5	2.791	4.265	5.793	7.366	8.973
3	3.316	4.897	6.515	8.174	9.866

Imax (A)

α \ β	1	1.5	2	2.5	3
1.5	475	441.5	423.8	413.4	406.9
2	322.2	287.47	269.6	259.5	253
2.5	258.5	223.46	205.6	195.2	188.6
3	222.9	188.17	170.34	159.8	153.1

Bmax (T)

α \ β	1	1.5	2	2.5	3
1.5	6	6.26	6.4	6.482	6.535
2	7.25	7.562	7.726	7.823	7.885
2.5	7.83	8.17	8.352	8.46	8.528
3	8.19	8.53	8.721	8.835	8.91

Bcent

α \ β	1	1.5	2	2.5	3
1.5	5.135	5.853	6.196	6.334	6.475
2	6.25	7.042	7.447	7.666	7.793
2.5	6.83	7.615	8.034	8.27	8.412
3	7.2	7.97	8.385	8.628	8.777

Vcoil (cm^3)

α \ β	1	1.5	2	2.5	3
1.5	122.7	184.1	245.4	306.8	368.2
2	294.5	441.8	589	736.3	883.6
2.5	515.4	773.1	1031	1289	1546
3	785.4	1178	1571	1963	2356

a = 25 mm, k = 0.72

FI (β = 2.6)

α \ β	1	1.5	2	2.5	3
1.5	1.61	2.7	3.83	5	6.18
2	2.474	3.91	5.41	6.952	
2.5	3.11	4.7	6.35		
3	3.66	5.33			

Imax (A)

α \ β	1	1.5	2	2.5	3
1.5	405.3	374.9	359	349.7	343.8
2	268.9	238.5	223.2	214.3	
2.5	213.5	182.5	168.3		
3	183	153.6			

Bmax (T)

α \ β	1	1.5	2	2.5	3
1.5	6.55	6.8	6.93	7.01	7.06
2	7.73	8.02	8.18	8.26	
2.5	8.27	8.58	8.74		
3	8.59	8.9			

Bcent

α \ β	1	1.5	2	2.5	3
1.5	5.6	6.36	6.71	6.9	7
2	6.67	7.47	7.88	8.1	
2.5	7.21	8	8.41		
3	7.56	8.32			

Vcoil (cm^3)

α \ β	1	1.5	2	2.5	3
1.5	122.7	184.1	245.4	306.8	368.2
2	294.5	441.8	589	736.3	
2.5	515.4	773.1	1031		
3	785.4	1178			

Fig. 5. Results of solenoid coil analysis

The analysis was made for two different compaction factors: k = 0.57 and k = 0.72. These factors correspond to two different coil winding patterns. The first one uses a thick insulating layer on the top of each wound layer of wire, so that good winding regularity is supported through the whole winding process. The second pattern uses thin insulation between the layers, so that during winding wire of the next layer finds its rest between the two next turns of the previous layer. Here higher compaction factors are expected, but also winding irregularity will show due to periodical wire jump that happens ones for each turn.

Vertical line β = 0.26 in Fig. 5 provides reference mark of maximal allowed length of the solenoid (according to specification). This position of the line takes into the account built of the length due to additional space required by cryostat (~ 50 mm), but does not take into the account other design features like flanges and bellows, that can appear in the final design.

Solid curves and colored zones in Fig. 5 correspond to combination of parameters that provide needed focusing strength of the lens in MEBT section. With about 40% of margin in field (or current) this corresponds to IF = 5 T^2m. It is possible to see that increasing compaction factor by ~ 25%, length of the coil can be reduced by ~ 25 mm, which can be important if one is trying to place focusing solenoids as close to each other as possible.

Other findings that can be mentioned are:

- volume of the coil increase sharply as one move towards higher α;
- center and maximal field also increase with α. A concern can be some increase in quench propagation velocity that will make protection more difficult.
- increasing α and decreasing β results in lower critical current.

IV. Summary

Design shown in Fig. 1 was developed using magnetic modeling as an instrument. In this design a = 28 mm, b = 48 mm, 2L = 125 mm, so α = 1.714 and β = 2.23. By using these input parameters with k = 0.56, we obtain: V_{coil} = 604 cm^3, B_{cent} = 7.21 T, B_{max} = 7.39 T, I_{max} = 306 A, and FI = 5.57 T^2m, which is within ~1% of what was found earlier by using magnetic modeling program. So, quasi-analytical approach gives accurate numbers combined with better performance (to fill the tables shown above (~ 50 runs) took about 3 hours).

This approach can be a useful instrument in choosing initial set of solenoid parameters.

V. References:

1. I. Terechkine, “Short Solenoid Lens Focusing Channel for PD Front End”, FNAL TD Note # TD-05-035.
2. P. Lee, ASC web site: http://www.asc.wisc.edu/plot/nb-ti_progress42.pdf
3. H. Knoepfel, “Pulsed High Magnetic Fields”, North Holland Publishing Company, Amsterdam, 1970.

VI Attachment: **Mathcad script. <u>Click to start working</u>**

Field Distribution for a Thick Solenoid

$\alpha := 1.714$ $\beta := 2.26$

$a := 28\cdot10^{-3}$ Inner radius

$b := \alpha\cdot a$ Outer radius

$b = 0.048$

$L := \beta\cdot a$ Half of the length

$L = 0.063$

<u>Wire parameters</u>

$d := 0.808\cdot10^{-3}$ bare wire

$di := 0.9\cdot10^{-3}$ insulated wire

$t := 0.1\cdot10^{-3}$ interlayer insulatioin thickness

$k := 0.56$ Compaction factor

$Sb := \frac{\pi\cdot d^2}{4}$ Bare wire cross-section

$Scoil := 2\cdot a^2\cdot(\alpha - 1)\cdot\beta$

$Vcoil := 2\cdot\pi\cdot a^3\cdot\left(\alpha^2 - 1\right)\cdot\beta$

$Vcoil = 6.04\cdot10^{-4}$

$Sw := \frac{Sb}{k}$ Effective cross-section of insulated wire in the coil

$N := \frac{Scoil}{Sw}$ number of turns

$N = 2.763\cdot10^{3}$

$I := 200$ Sample wire current (A). Used to define later required current to reach quench